Exam 3

$\boxed{1}$ YES (A)

$$I_{DISK} = \frac{1}{2} m_D r^2 \qquad I_{RING} = m_R r^2$$

$$I_D = I_R \quad \text{if} \quad m_D = 2 m_R \qquad (\text{given } r_D = r_R = r)$$

Disk is more massive than Ring

$\boxed{2}$ YES (B)

$$I_{DISK} = \frac{1}{2} m r_D^2 \qquad I_{sphere} = \frac{2}{3} m r_S^2$$

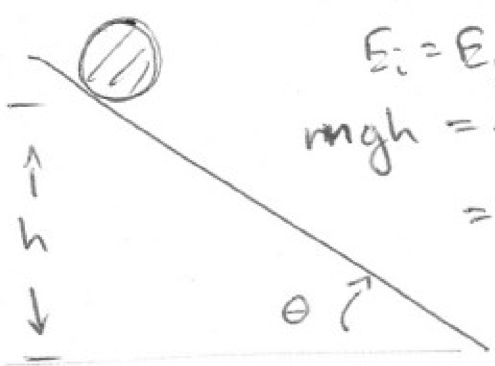
$$I_D = I_S \quad \text{if} \quad r_D = \frac{2}{\sqrt{3}} r_S \qquad (\text{given } m_D = m_S = m)$$

$$\frac{1}{2} m r_D^2 = \frac{2}{3} m r_S^2$$

$$r_D = \frac{2}{\sqrt{3}} r_S$$

Disk has larger diameter than Sphere

$\boxed{3}$



$$E_i = E_f$$

$$mgh = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2 \qquad I = \frac{1}{2} m r^2 \qquad v = r\omega$$

$$= \frac{1}{2} m v^2 + \frac{1}{2}\left(\frac{1}{2} m r^2\right)\omega^2 = \frac{1}{2} m v^2 + \frac{1}{4} m (r^2 \omega^2)$$

$$= \frac{1}{2} m v^2 + \frac{1}{4} m v^2 = \frac{3}{4} m v^2$$

$$\cancel{m} gh = \frac{3}{4} \cancel{m} v^2$$

$$\left(\frac{4gh}{3}\right)^{1/2} = v$$

*velocity is independent of mass or diameter

They reach bottom at same time. (C)

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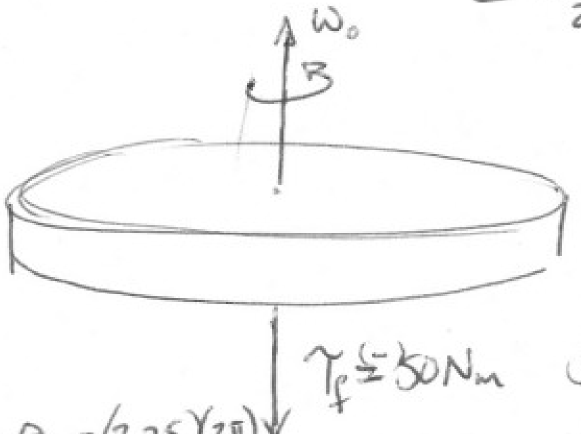
$$I = \frac{1}{2}mr^2 = \frac{1}{2}(50\,kg)(1\,m)^2 = 25\,kg\,m^2$$



$$\sum \tau = I\alpha$$

$$-\tau_f = I\alpha \qquad \alpha = \frac{-\tau_f}{I}$$

$$\theta - \theta_0 = \underset{rev}{(2.25)}\underset{rad/rev}{(2\pi)}$$

① $\omega = \int \alpha\, dt = \omega_0 + \alpha t$

② $\theta = \int \omega\, dt = \theta_0 + \omega_0 t + \alpha \frac{t^2}{2}$

① $\omega = 0$ rad/sec $\qquad -\omega_0 = \alpha t = \frac{-\tau_f}{I} t \qquad t = \frac{I\omega_0}{\tau_f}$

② $\Delta\theta = 2.25(2\pi)$ rad $\qquad \Delta\theta = \omega_0 t + \frac{-\tau_f}{I}\frac{t^2}{2}$

$$\Delta\theta = \omega_0\left(\frac{I\omega_0}{\tau_f}\right) - \frac{\tau_f}{I}\frac{1}{2}\left(\frac{I\omega_0}{\tau_f}\right)^2 = \omega_0^2\left(\frac{I}{\tau_f} - \frac{I}{2\tau_f}\right)$$

$$\Delta\theta = \omega_0^2 \frac{I}{2\tau_f} \quad \text{so} \quad \omega_0 = \left(\frac{\Delta\theta\, 2\tau_f}{I}\right)^{1/2}$$

$$\omega_0 = \left(\frac{2.25\cdot(2\pi)\,2\,(50\,Nm)}{25\,kgm^2}\right)^{1/2}$$

$$\omega_0 = 7.5\,\frac{rad}{sec}$$

Energy initial is $\frac{1}{2}I\omega_0^2$ $\quad$ (or $\int \tau\, d\theta = \tau_f \Delta\theta$)

$$\frac{1}{2}(25\,kgm^2)\left(7.5\,\frac{rad}{sec}\right)^2 = 707\,J \qquad\qquad = 707\,J$$

units ✓

$\tau_f \uparrow \quad \omega_0 \uparrow$ (need more initial speed to overcome friction and rotate 2+)

$I \uparrow \quad \omega_0 \downarrow$ less energy if I is smaller

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If merry-go-round has no other $\vec{\tau}$ then $\vec{\alpha}$ is along (same direction) as $\vec{\tau}_f$ (it is slowing down). If $\Sigma \vec{\tau} = 0$ then $\vec{\alpha} = 0$

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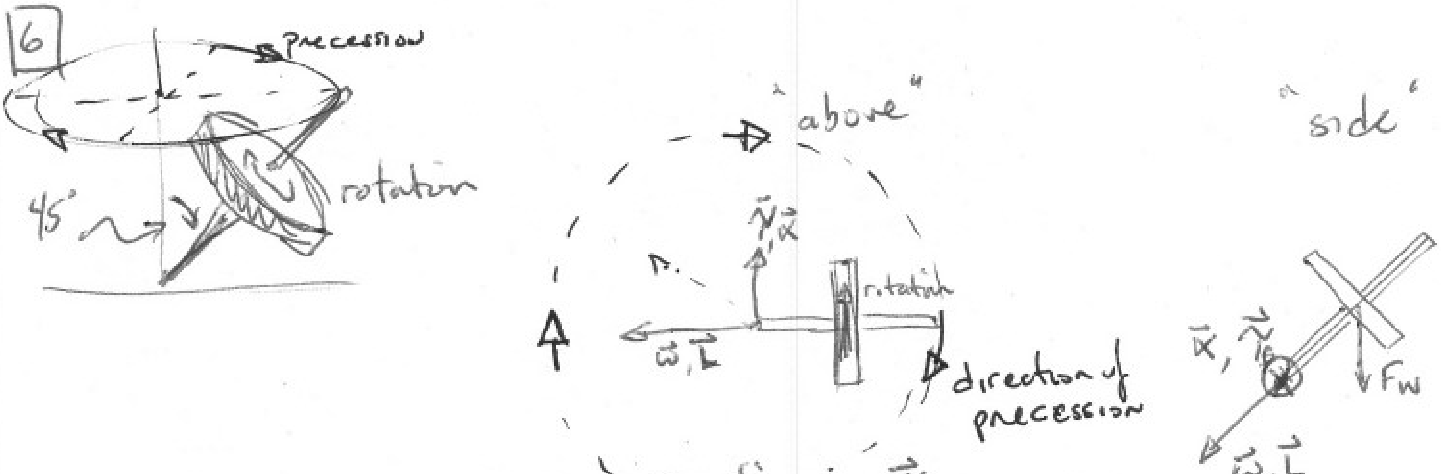

$\vec{\alpha}$ is along same ~~direction~~ as $\vec{\tau}$
($\vec{\omega}$ is changing direction)

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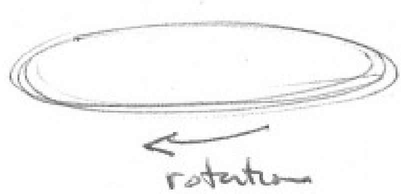
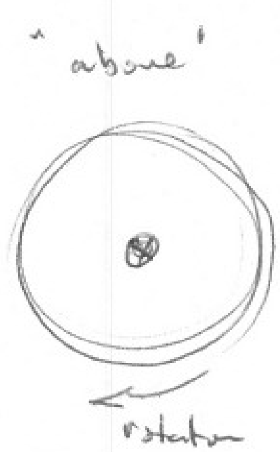
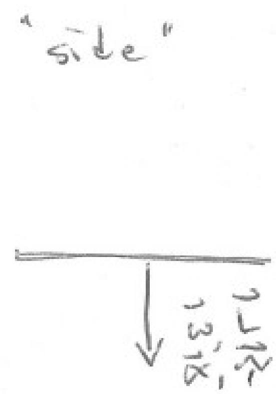


record is speeding-up.





all along same direction